Understanding the radar equation

free training document

Dipl.-Ing. (FH) Christian Wolff,

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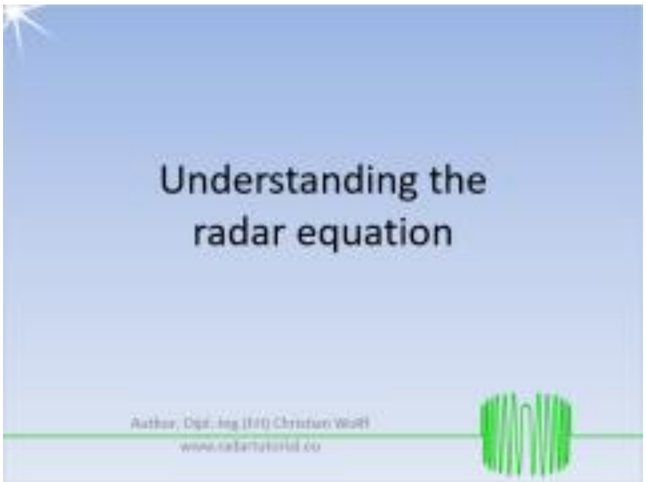


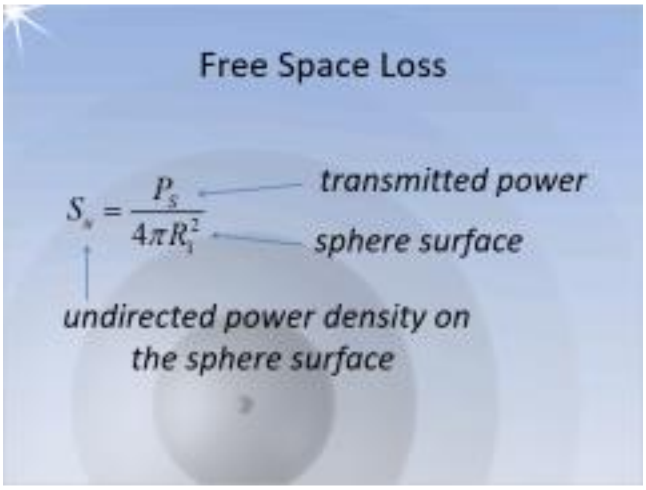
Statement: *"The radar equation represents the physical relationships from the transmission power to the wave propagation, including the reflection and the way back to the reception. "*

Question of a physicist: Why transmission **power**, the wave propagation is really an **energy** transport?

Answer: Power is defined as energy per time unit. Assuming that the duration of the transmitted signal is not, or not significantly, different from the duration of the reflected and received signal, then the ratio of transmitted energy to received energy can be replaced by the ratio of the transmitted power to the received power as the unit of time is shortened mathematically. Power is easier to measure than energy in radar, which is why a power ratio is preferred.

In this presentation, the radar equation for a surveillance radar and point-like targets is derived.

In the following, we will first assume that electromagnetic waves can propagate under ideal conditions, i.e. without interference.

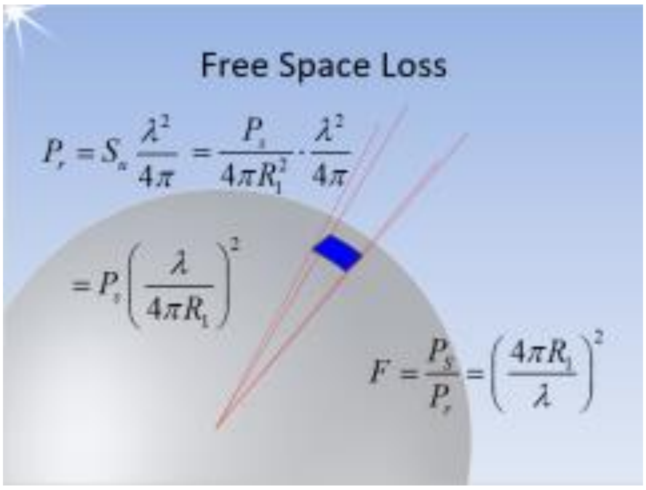


We first consider what happens to the transmitting power when it is radiated into free space. Before we consider the directivity and antenna gain of a real antenna, let's assume that the transmitter radiates isotropically, i.e. evenly in each direction without preferring any of the directions.

Thus, the transmitting power is distributed spherically around the transmitter. Since the electromagnetic waves propagate at approximately the speed of light, the spherical surface becomes larger and larger. The power density per unit area on this spherical surface results from the ratio of a small unit area to the total surface area of the sphere, multiplied by the transmitting power.

Here, the transmit power is still undirected, hence the symbol S_u. The expression in the denominator is the sphere surface, where R is the radius of the sphere - it is also at the same time the previous distance (called range in radar jargon) from the small area unit, which will later be replaced by the reflecting object. The index R_1 is only the outward path for the time being. As usual for power densities, the unit of measurement is given by watts per square meter, which results from the transmit power divided by an area (here: the square of the distance).

We assume that the transmit power is constant over the period under consideration. That is, the only variable left in this equation is distance. From this, we can deduce that the power density at the receiving location decreases with the square of the distance. This corresponds to the inverse-square law known in physics.



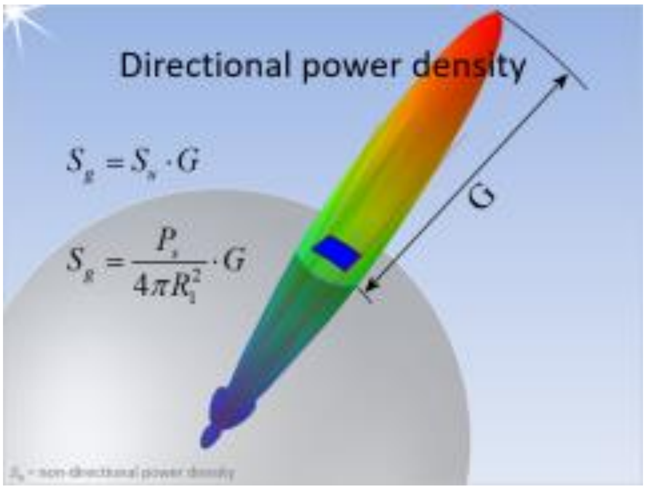
A second aspect of the free space loss is that this considered section from the sphere surface cannot be made arbitrarily small. A receiving antenna always has an effective area: the aperture. The smallest possible aperture of a receiving antenna depends on the transmitted wavelength: here width and height, i.e. λ^2. The maximum number of such squares on the spherical surface depends on their size concerning the spherical surface, i.e. λ^2 divided by 4π. (The radius here is still the unit circle with radius r=1). Thus, the free space loss becomes frequency-dependent!

We have already seen the undirected power density S_u as the transmitted power distributed on the surface of the sphere, and we use this expression here.

We summarize the quantities to be squared.

Up to this point, however, it is still a gain factor much smaller than 1 by which the transmit power is multiplied.

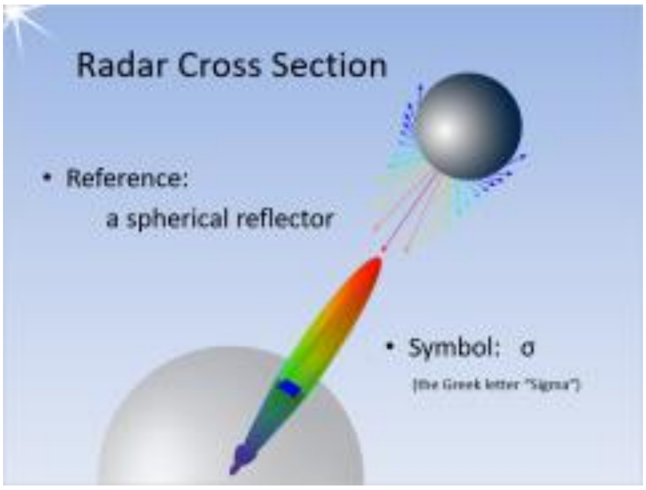
The free space loss F is then the reciprocal of this gain, the ratio of transmit power to receive power. A free space loss is thus just a dimensionless number.



Let's return to the power density. Now we use a radar antenna with strong directivity. Compared to an isotropic radiator, this antenna has an antenna gain G. The undirected power density now becomes the directed power density S_g. Thus, the directional power density, increased by the antenna gain, arrives at the reflection object.

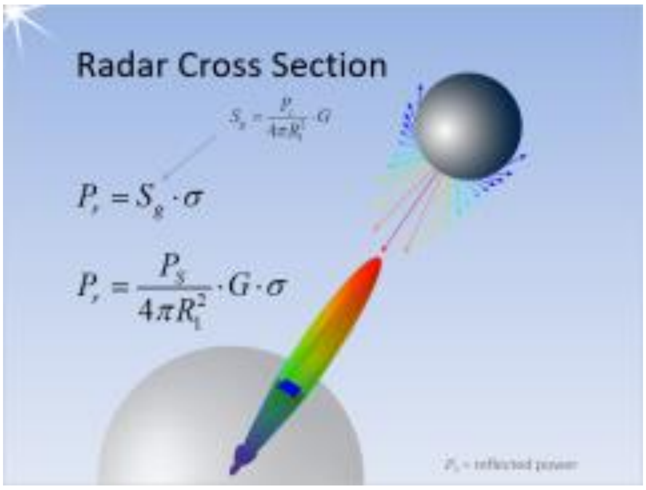
We have already learned about the undirected power density as the ratio of the transmitted power to the sphere surface and can replace this term here.

Now it depends on how large the effective reflection area of this object is, i.e. how much of this power density is backscattered.



This effective reflection area (also called Radar Cross Section, RCS) can assume very different values: several hundred square meters for a large container ship down to fractions of square centimeters for a plastic projectile from a small shotgun. The spherical reflector is used as a reference, which in turn causes an isotropic, i.e. an equally strong reflection, regardless of the direction from which it is illuminated. The visible area should be 1 m² from all directions, so the diameter of the sphere must be about 1.3 m.

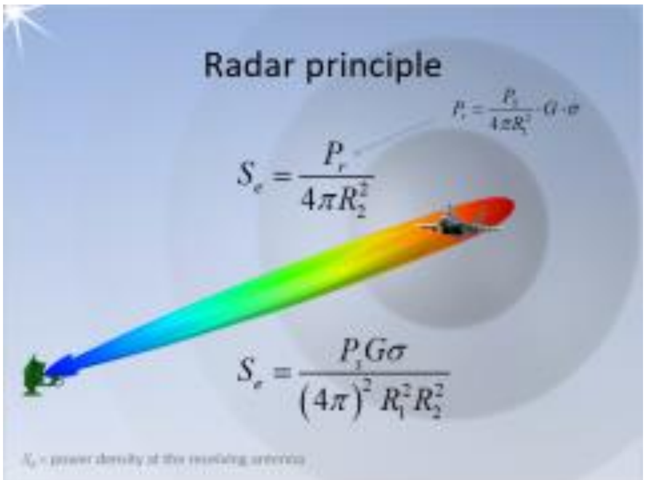
The reflected power P_r is now dependent on the power density S_g existing at the location of reflection, and the size of the reflector, i.e. its radar cross section (sigma).



The small partial area on the surface of the sphere considered so far is now given a variable size and multiplied by the power density at this location. Thus, the unit of measurement square meter is truncated again and a power variable is created: the reflected power P_r in watts.

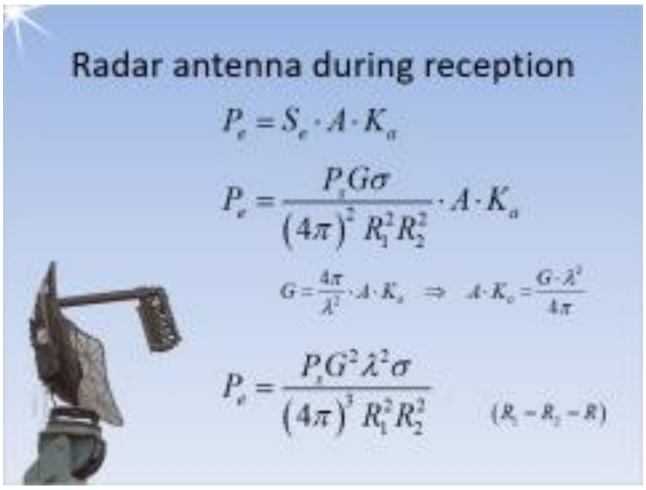
Simplified: the reflecting object can again be considered a radiator based on the reflected power. The reflected power then becomes the radiated power (from the reflector).

On the return path of the echo signals, the same conditions prevail as on the outward path. Also, here the power density at the radar antenna *Se* is only a partial area of a sphere surface with the radius R_2 (here: of the return path).



We have derived in the previous slides, how large the reflected power is concerning the transmitted power, and we can insert this term here.

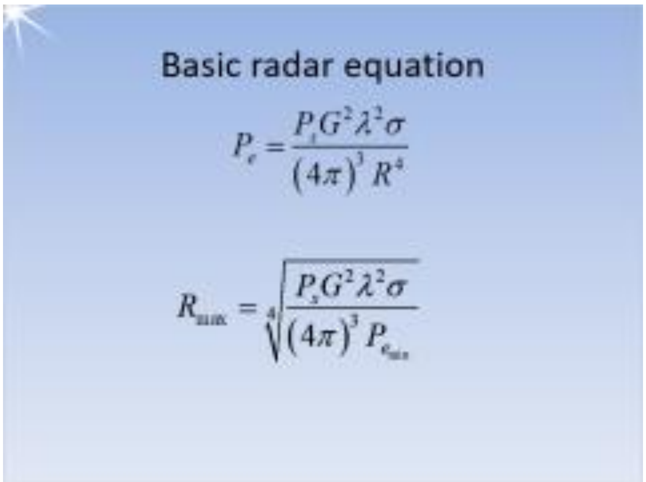
The receiving antenna of the radar is located in the environment of this power density. This antenna also has an effective antenna area, which is formed by the geometric area A multiplied by a matching factor K_a containing the efficiency. Together with the power density Se applied at this location, this gives the received power of the antenna P_e.



We replace the variable S_e with the expression already derived.

However, the effective antenna area is a rather unwieldy value here. This value also appears in another equation that arises when deriving the antenna gain of an antenna. If this equation is rearranged to $A \cdot K_a$, we can substitute this term. As a result, the 4π now appears to the third power and the wavelength lambda appears in the equation. The antenna gain G is now raised to the square: this is also logical because this gain acts on both transmission and reception.

For a monostatic radar using the same antennas for transmitting and receiving, the outward path R_1 is equal to the return path R_2 and can be further summarized as R^4.



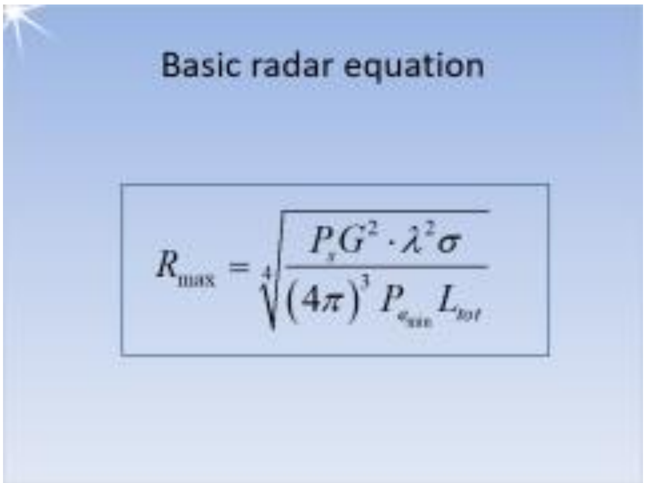
We can rearrange this equation in the first step according to the distance. This creates the well-known equation with a long fraction line under the fourth root.

In a second step, we assume that the received power should be equal to the minimum possible received power.

This gives us an equation that determines the theoretical maximum possible range of a radar.

So far, however, we have assumed ideal conditions without internal or external additional losses, such as attenuation on feed lines, in the case of losses in the matching of the antenna, or the conversion of conducted waves to space waves, and during the propagation of the electromagnetic waves.

These losses can all be summed up to a loss factor L_{tot}. Thus, the **basic** radar equation is completely derived.



This equation is independent of the modulation type and thus universally applicable for every reconnaissance radar. However, different radars have different loss quantities. In some circumstances, these can even become gains, as in the case of a chirp radar using pulse compression, where the duration of the transmitted signal and the duration of the compressed echo signal are just not equal. Similarly, pulse integration, i.e., improving the signal-to-noise ratio over multiple pulse periods, can be used because the noise is rarely synchronous with the echo signal and therefore a sum of the voltages does not amplify the noise to the same degree as the echo signal.

However, larger differences in the application of this equation arise in the case of a weather radar, where volume targets are located rather than point targets. In this case, the size of the effective reflection area σ is also dependent on the distance, which changes the structure of the entire equation.

What can be done with this basic radar equation? Certain parameters can be influenced only a little or not at all by the user of a radar unit, they are given by the manufacturer. These would be for example the size of the antenna and thus the antenna gain and the used wavelength. Also, the transmit power and the receiver sensitivity can often only be influenced to a small extent. However, losses in the feed lines can be minimized by good maintenance.

In contrast, the value of the radar cross section is very variable. It is the reason why targets with a very small radar cross section are very difficult to detect. The range of the radar is very limited here despite a good design concept and a good maintenance condition.

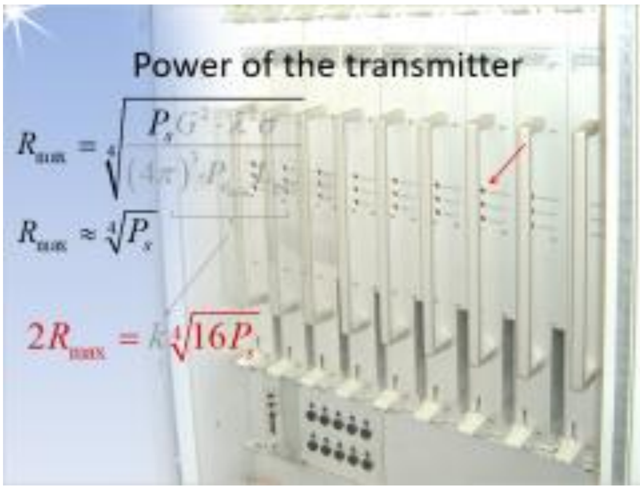
From this basic radar equation, already essential properties of radar can be derived. If, for example, the wavelength of the radar is reduced, i.e. the transmission frequency is increased, then the range is reduced! Radar units with a large range in air defense therefore usually work with a lower transmission frequency. The ideal would be a radar in the VHF to UHF range because here the electromagnetic waves still propagate reasonably straight.

But the geometrical size of the antenna is also hidden in the antenna gain: see again the equation with the antenna gain: For a desired (constant) antenna gain, the size of the antenna is proportional to the wavelength. That means: if the antenna gain is the same, the necessary geometrical extension of the antenna will also be larger with a larger wavelength! The picture shows a VHF/UHF radar with a parabolic reflector with a diameter of 46 m.

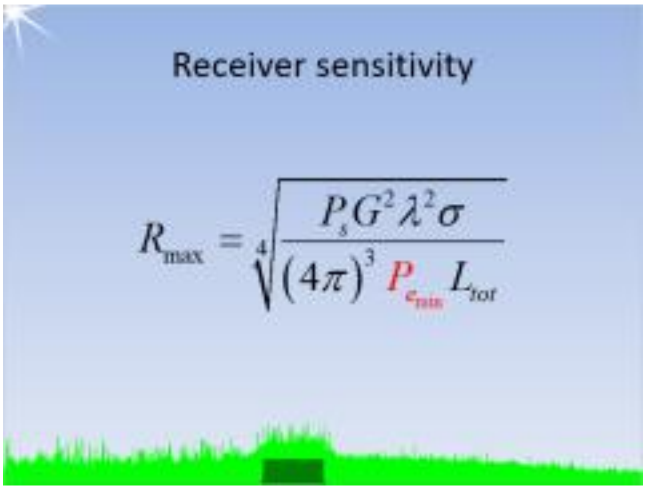
A compromise between antenna size, range, and angular accuracy is the L-band, in which most long-range radars operate. This applies to both air defense and air traffic control: en-route radars operate at frequencies between 1.25 and 1.35 GHz. In the picture the air traffic control radar in the gray North has a parabolic reflector of 9 by 14 meters, so it is already much smaller than the previous example.

However, a large wavelength also has a disadvantage. The possible accuracy of radar is also a function of the wavelength. Therefore, radars with higher accuracy requirements operate at higher frequencies, despite any limitations in range.

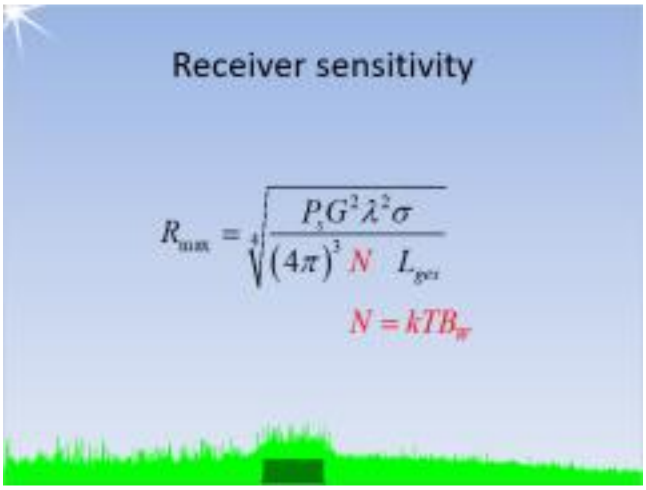
It is obvious that a higher transmitting power also results in a higher range. However, this is not a linear relationship. The transmitting power is under the fourth root. For clarity, let's summarize the other variables as a constant factor.
To double the range, the transmit power would have to be increased sixteenfold!
A small increase or decrease of the transmit power has almost no measurable influence on the range of a radar. A 10% reduction in transmit power causes a range loss of about 2.6% (the fourth root of 0.9 is 0.974). The picture shows power amplifier modules of the transmitter of an air traffic control radar. The power amplifier stage consists of up to 32 such modules. If one of these modules fails, a remaining range of 99.2% (the fourth root of 31/32) remains. This would not be noticeable on the radar display. But the technician sees that the red LEDs “module error” and “module off” light up and can change this module even during operation.



Also, to screw at the sensitivity of the receiver does not bring substantially much. Something in the order of 16x would also have to be done here (that would be +12 dB). But the sensitivity of the receiver is strongly dependent on the noise level. If the received power does not exceed the noise level, then this echo signal will not be detected. One can integrate this relationship immediately into the radar equation (However, I call it no more radar **basic** equation now!).



Here you must know, however, that an echo pulse does not lie before, behind, or between the noise, but that the voltage of the echo signal and the voltage of the noise pulses add up. The echo signal, therefore, “pushes” the noise upwards. In practice, however, this is only possible with well-tuned analog displays used by an experienced operator who can still detect many a faint target “under” the noise. Technical circuits, on the other hand, would not detect an echo signal until it is at least twice as large as the noise level.

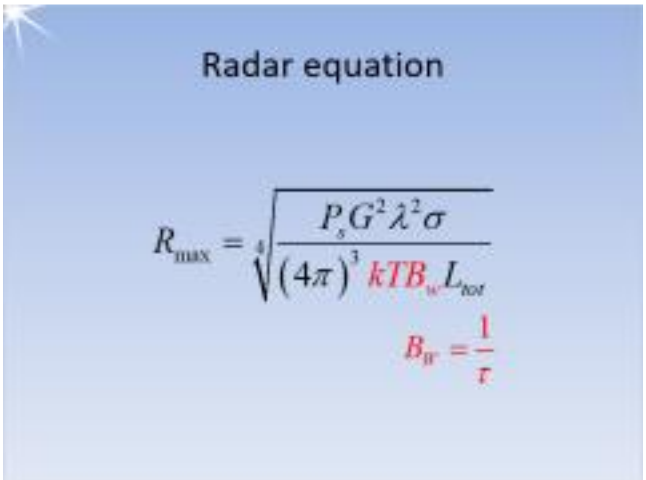


The basic idea is that the received power must be greater than the noise power at the receiver input. The limiting case is that it is equal. You can equate the P_{Emin} with the noise power N (this variable name is derived from Noise).
The reason for this noise is the thermal motion of all particles when exposed to heat. To the same extent, the electrons also resonate. This causes a basic noise in every cable, every resistor, and every semiconductor component. The higher the temperature, the higher the noise.
At a temperature close to zero (-273°C), the noise is also close to zero.

$$N = kTB_W$$

where: k the Boltzmann constant k= 1.38 x 10-23 J/K,
T the temperature in Kelvin and
B_W the bandwidth of the receiver.

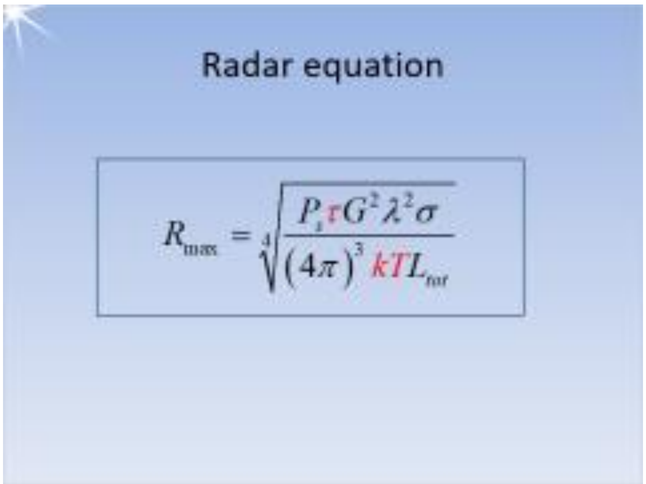
Usually, the ambient temperature T_0 = 290 K is chosen as the reference temperature. This corresponds to about 20° C. (click)
We replace the P_{Emin} with the term kTB_W ...



The influence of the receiver bandwidth arises from the fact that the so-called white noise is extremely broadband. The narrower the bandwidth of the receiver, the less interfering noise it can receive. But the bandwidth of the receiver should be at least as large as the bandwidth of the transmitter to receive its echo signals without loss.

This transmitter bandwidth depends on the transmission pulse duration for classical pulse radar. The variable τ (the Greek letter “Tau”) here is the duration of the transmit pulse.

We replace the bandwidth with the transmit pulse duration (tau).
The transmit pulse duration is usually placed right next to the transmit power. This better demonstrates that the range depends on the transmit energy - not ostensibly on the transmit power. This is also the reason why a continuous wave radar needs much less transmit power compared to a pulse radar to get passable ranges.
And by the way, we have also confirmed the physicists' remark made at the beginning that wave propagation is an energy transport.
Because transmitting power is energy per unit of time. If we multiply the transmission power with a time, the time unit is shortened from the power and the transmission energy remains. Nevertheless, we leave the term transmission power here, because electric power can be measured much better with known pulse duration, than energy.



You can read this whole course of the derivation on the radar tutorial and print it out if necessary.

This page can be found in the section Basics under the entry Radar Equation.